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CLASS 12TH - Ch-3

MATRICES

WHAT IS A MATRICE

- **Arrangement of information** in the form of a **RECTANGLE** or **SQUARE** expression is called as Matrice.

- This arrangement is done in **ROWS and COLUMNS** which described the **ORDER** of the Matrice.

- It is denoted by **A** normally. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ — ROW, COLUMN

R*C
3*3

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

R*C
2*3

EXAMPLE :

MARKS OF 5 SUBJECTS OF 10 STUDENTS (A-J)
are expressed in the form of A MATRICE as given below:

	A	B	C	D	E	F	G	H	I	J
ENG	100	85	75	90	100	85	95	70	70	80
HINDI	85	80	60	95	85	65	65	65	80	80
MATHS	80	85	75	100	65	100	95	75	70	90
SCIENCE	90	100	80	90	90	100	80	90	80	80
SST	100	100	90	80	70	90	90	80	70	100

Here Subject wise marks are expressed in Rows(Horizontally) and the Student No. are presented in Columns (Vertically)

EXAMPLE :

MARKS OF 5 SUBJECTS OF 10 STUDENTS
are expressed in the form of A MATRICE as given below:

	A	B	C	D	E	F	G	H	I	J	
ENG	100	85	75	90	100	85	95	70	70	80	ROW -1
HINDI	85	80	60	95	85	65	65	65	80	80	
MATHS	80	85	75	100	65	100	95	75	70	90	ROW -3
SCIENCE	90	100	80	90	90	100	80	90	80	80	
SST	100	100	90	80	70	90	90	80	70	100	ROW -5
	C-1		C-3		C-5		C-7			C-10	

ORDER:-
R*C = 5*10

MARKS OF 5 SUBJECTS OF 10 STUDENTS
are expressed in the form of A MATRICE as given below:

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	
ENG	100	85	75	90	100	85	95	70	70	80	**ROW -1**
HINDI	85	80	60	95	85	65	65	65	80	80	
MATHS	80	85	75	100	65	100	95	75	70	90	**ROW -3**
SCIENCE	90	100	80	90	90	100	80	90	80	80	
SST	100	100	90	80	70	90	90	80	70	100	**ROW -5**
	C-1		**C-3**		**C-5**		**C-7**			**C-10**	

The above arrangement with the help of Matrice helps in understanding the above information in the shortest period of time and so various decision can be made through that information.

In Matrice, the information is expressed in the format of Rows and Columns.

Rows are expressed Horizontally and Columns Vertically and the rows and columns together determine the order of a Matrice.

What's the order?

The order is something which explains the Matrice. For Ex. A 2*3 Matrice means there are 2 rows and 3 columns. A 3*4 Matrice means there are 3 rows and 4 columns.

Rows, Column, Elements and Order of Matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

ROW:
R1
R2
R3

COLUMN:
C1, C2, C3

ELEMENTS

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\mathbf{3 * 3}$$

ADVANTAGES OF MATRICES:

- Through MATRICE, we can read the vast information easily, in a short period of time.
- In this chapter, we will solve the **system of linear equations through Matrice.**
- Also, a lot of analysis can be made through the information presented in the form of MATRICE.
- For EG.In the example explained, We can have student wise and subject wise marks. We can evaluate anything that is required.

MATRICE is used in various other fields like :-

- Matrices are used in representing the real world data's like the population of people, infant mortality rate, etc.,
- Graphic software such as Adobe Photoshop uses matrices to process linear transformations to render images.
- Matrices are used in the study of electrical circuits, quantum mechanics and optics.
- In hospitals, medical imaging, CAT scans and MRI's, use matrices to operate.

TYPES OF MATRICES

1. SQUARE MATRICE:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

No of columns = No. of rows

i.e. 2×2, 3×3,4×4, 5×5.

2. RECTANGLE MATRICE

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

No of columns ≠ No. of rows

i.e. 2×3, 3×2,4×1, 1×5.

3. DIAGONAL MATRICE

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Non diagonal elements should be ZERO*

*There is only 1 diagonal which starts from Top R1 to Bottom of R3 (Left-Right)

4. SCALAR MATRICE

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Non diagonal elements should be ZERO and Diagonal element shall be a same constant.

5. IDENTITY MATRICE

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Non diagonal elements should be ZERO and Diagonal element shall be 1

6. NULL MATRICE

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

ALL elements should be ZERO

7. ROW MATRICE

$$[1\ 2\ 3]$$

Only one Row

8. COLUMN MATRICE

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Only one Column

ADDITION OF MATRICES

IF $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ IF $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ THEN $A + B = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$

IF $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ IF $B = \begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{bmatrix}$ THEN $A + B = \begin{bmatrix} 11 & 22 & 33 \\ 44 & 55 & 66 \\ 77 & 88 & 99 \end{bmatrix}$

- For Addition and Subtraction; the order of both the matrices should be the SAME.
- Matrix Addition / Subtraction is very simple; we just add the corresponding elements.

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 8 \\ 4 & 8 \\ 9 & 6 \end{bmatrix}$$

Order is different, we cannot add or subtract.

$$A = \begin{bmatrix} 9 & -6 & 3 \\ 2 & -8 & 5 \\ 2 & -4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} -5 & 3 \\ -2 & 1 \end{bmatrix}$$

Order is different, we cannot add or subtract.

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} -5 & 3 \\ -2 & 1 \end{bmatrix}$$

Order is same, we can add or subtract.

SUBTRCTION OF MATRICES

IF A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ IF B = $\begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ -70 & -80 & -90 \end{bmatrix}$ THEN A- B = $\begin{bmatrix} -9 & -18 & -27 \\ -36 & -45 & -54 \\ 77 & 88 & 99 \end{bmatrix}$

IF A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ IF B = $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ THEN A- B NOT POSSIBLE

as order of both the matrices are different

WHAT IS KA

IT MEANS MULTIPLYING EVERY ELEMENT OF MATRICE BY K

IF A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ THEN 3A = $\begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$

IF A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ THEN 5A = $\begin{bmatrix} 5 & 10 & 15 \\ 20 & 25 & 30 \\ 35 & 40 & 45 \end{bmatrix}$

A'= Transpose of a Matrice

Through Transpose : Rows of a Matrice are converted into column and the columns are converted into rows.

If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ Then A' = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

Here R 1 is converted into C 1 and R 2 is converted into C 2

If A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ Then A' = $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

MULTIPLICATION OF 2 MATRICES

IF A = $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ & B = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

THEN A × B = $\begin{bmatrix} A \times 1 + B \times 3 & A \times 2 + B \times 4 \\ C \times 1 + D \times 3 & C \times 2 + D \times 4 \end{bmatrix}$

i.e. $\begin{bmatrix} R1 \times C1 & R1 \times C2 \\ R2 \times C1 & R2 \times C2 \end{bmatrix}$

MULTIPLICATION OF 2 MATRICES

IF $A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

THEN $A \times B = \begin{bmatrix} 5 \times 1 + 6 \times 3 & 5 \times 2 + 6 \times 4 \\ 7 \times 1 + 8 \times 3 & 7 \times 2 + 8 \times 4 \end{bmatrix}$

$$\begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$$

MULTIPLICATION OF 2 MATRICES

IF $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ & $B = \begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{bmatrix}$

THEN $A \times B = \begin{bmatrix} R1 \times C1 & R1 \times C2 & R1 \times C3 \\ R2 \times C1 & R2 \times C2 & R2 \times C3 \\ R3 \times C1 & R3 \times C2 & R3 \times C3 \end{bmatrix}$

$$A \times B = \begin{bmatrix} 10 + 80 + 210 & 20 + 100 + 240 & 30 + 120 + 270 \\ 40 + 200 + 420 & 80 + 250 + 480 & 120 + 300 + 540 \\ 70 + 320 + 630 & 140 + 400 + 720 & 210 + 480 + 810 \end{bmatrix} = \begin{bmatrix} 300 & 360 & 420 \\ 660 & 810 & 960 \\ 1020 & 1260 & 1500 \end{bmatrix}$$

MULTIPLICATION OF 2 MATRICES

IN THE LAST EXAMPLES, BOTH THE MATRICES ARE OF SAME ORDER SO THE RESULTANT MARICE COMES OUT TO BE OF THE SAME ORDER TOO.

NOW WHAT ABOUT WHEN THE **ORDER** OF TWO MATRICES ARE **DIFFERENT.**

EG:

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix} = \begin{bmatrix}1+8+21 & 2+10+24 & 3+12+27\\4+20+42 & 8+25+48 & 12+30+54\end{bmatrix} = \begin{bmatrix}30 & 36 & 42\\66 & 81 & 96\end{bmatrix}$$

2*3   3* 3   2*3

SAME

*Two matrices can be multiplied ONLY
When the C of first matrice and the R of second matrice are the same.

And the resultant matrice will have the order of R of first matrice and C of second matrice as clear from the above.

MULTIPLICATION OF 2 MATRICES

Exercise 3.2 Q3(ii) $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}2 & 3 & 4\end{bmatrix}$

IF A = $\begin{bmatrix}1\\2\\3\end{bmatrix}$ **& B =** $\begin{bmatrix}1 & 2 & 3\end{bmatrix}$

3×1 $\quad$ 1×3

THEN A×B = $\begin{bmatrix}R1\times C1 & R1\times C2 & R1\times C3\\R2\times C1 & R2\times C2 & R2\times C3\\R3\times C1 & R3\times C2 & R3\times C3\end{bmatrix} = \begin{bmatrix}1\times1 & 1\times2 & 1\times3\\2\times1 & 2\times2 & 2\times3\\3\times1 & 3\times2 & 3\times3\end{bmatrix} = \begin{bmatrix}1 & 2 & 3\\2 & 4 & 6\\3 & 6 & 9\end{bmatrix}$

MULTIPLICATION OF 2 MATRICES

If $B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1 \times 3}$ & $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3 \times 1}$

then $B \times A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = (1\times1 + 2\times2 + 3\times3) = (14)$

MULTIPLICATION OF 2 MATRICES

Exercise 3.2

Q3(v)

IF $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

3×2 $\quad$ 2×3

THEN $A \times B = \begin{bmatrix} R1\times C1 & R1\times C2 & R1\times C3 \\ R2\times C1 & R2\times C2 & R2\times C3 \\ R3\times C1 & R3\times C2 & R3\times C3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

Elements:-

The elements of Matrices are denoted as:- $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

So if A = $\begin{bmatrix} 91 & 33 & 66 \\ 5 & 11 & 82 \\ 69 & 5 & 0 \end{bmatrix}$

Then

$a_{11} = 91, a_{12} = 33, a_{13} = 66$

$a_{21} = 5, a_{22} = 11, a_{23} = 82$

$a_{31} = 69, a_{32} = 5, a_{33} = 0$

EXERCISE (CW +HW)

Q.1. Find the sum of A and B where $A = \begin{bmatrix} 2 & 3 \\ -5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 2 & -11 \end{bmatrix}$.

Q.2. Find A +B when $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 5 & 11 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -2 & -3 \\ 5 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix}$.

Q.3. If $A = \begin{bmatrix} -1 & 2 & -3 \\ -2 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 & 2 \\ 3 & 0 & 1 \end{bmatrix}$, then find the sum of A and B.

Q.4. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 3 \\ 1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 & 0 \\ -2 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 0 & -3 \\ 1 & 1 & -1 \end{bmatrix}$, find A + B + C.

Q.5. If $A = \begin{bmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{bmatrix}$,$B = \begin{bmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{bmatrix}$ find 4A -3A.

Q.6. If $A = \begin{bmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{bmatrix}$ then Find 2A+B

Q.1. $\begin{bmatrix} 6 & 9 \\ -3 & -4 \end{bmatrix}$

Q.4. $\begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ 2 & 2 & 1 \end{bmatrix}$

Q.2. $\begin{bmatrix} 5 & 1 & 1 \\ 10 & 10 & 10 \\ 9 & 8 & 13 \end{bmatrix}$

Q.5. $\begin{bmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{bmatrix}$

Q.3. $\begin{bmatrix} -1 & 1 & -1 \\ 1 & 1 & 5 \end{bmatrix}$

Q.6. $\begin{bmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{bmatrix}$

Since **A,B** are of the same order **3x3**,subtraction of **4A** and **3B** is defined.

Q.5.

$$\mathbf{4A - 3B} = 4\begin{bmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{bmatrix} - 3\begin{bmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 16 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{bmatrix} + \begin{bmatrix} 21 & 12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{41} & \mathbf{4} & \mathbf{1} \\ \frac{\mathbf{5}}{\mathbf{4}} & -\frac{\mathbf{15}}{\mathbf{2}} & \mathbf{4\sqrt{2} - 9} \\ \mathbf{-11} & \mathbf{54} & \mathbf{-11} \end{bmatrix}$$

Since **A and B** have same order **3x3, 2A +B** is defined.

Q.6.

We have,$\mathbf{2A + B} = 2\begin{bmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{bmatrix} + \begin{bmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{bmatrix}$

$$= \begin{bmatrix} \mathbf{18} & \mathbf{27} & \mathbf{9} \\ \mathbf{1} & \mathbf{8} & \mathbf{22} \\ \mathbf{-11} & \mathbf{11} & \mathbf{-2} \end{bmatrix}$$